A SIMPLIFIED MODEL TO PREDICT CHANGES IN A CONFINED AQUIFER

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ABSTRACT

Some pressure sensitive reservoirs show sensitivity to many variables, including permeability and formation thickness. The latter phenomenon may lead to high cost problems at the surface.

The objective of the present study is to estimate the formation thickness as a function of fluid pressure. To reach this end, we derive algebraic equations to quantify thickness changes in an approximate way. The methodology depends on the assumptions of steady state flow and exponential variation with pressure of all pressure dependent variables. We use a well-known logarithmic pressure transform. Then the linearization of the diffusivity equation in terms of the transformed variable is complete, and analytical solutions are readily available. Solutions in terms of pressure are obtained by an inverse transform. Due to the non-linearity, the reservoir behaves different during production and injection. The thickness depends on the value of the corresponding elastic modulus but also on the total elastic modulus. The proposed methodology may be extended to time-dependent problems, but with reduced accuracy. Perturbations techniques are available to improve the accuracy.

The proposed methodology depends on favorable results from core analysis. Formation thickness may be evaluated as a function of porosity. Our technique may be used provided the variation of thickness with pressure follows an exponential function with acceptable accuracy (correlation coefficient).

New aspect:
Use of pressure transforms to account for multiple non-linearity in a new way. Provide an approximate assessment of simultaneous changes in formation thickness, permeability, density and well performance relationship.

Conclusion:
A composite elastic modulus is useful to account for multiple non-linearities. The effect of pressure changes on each variable single term may be estimated, thickness included.

Keywords: Compaction, Expansion, Deformable aquifer, Stress-sensitivity

1. Introduction

This study addresses interaction of pressure dependent variables. The theory depends on the assumption that all variables may be characterized by a constant elastic modulus over a limited pressure range. We think about the proposed methodology as a simplistic tool to be used when decisions has to be made based upon limited information. When additional information becomes available, numerical simulations are the preferred approach. For sake argument we assume that reasonable values for the permeability- and thickness moduli may be estimated from core analysis (Wyble, 1958; Jelmert and Selseng, 1998) and that the remaining values may be estimated by correlations and or laboratory studies.

Compaction may be classified as non-recoverable and or recoverable, (Helm, 1984). For shallow aquifers, compaction of the first type is usually dominant. The present technique deals with recoverable compaction. We investigate an elastic response in a deep aquifer. A hydrothermal reservoir could be an example. Another could be waste water disposal.

2. Theory

Raghavan *et al*. (1972) proposed a well test model for pressure-dependent rock and fluid properties which included thickness changes. They found:

$$h(p) = \frac{h_e\left(1-\varphi_e\right)}{1-\varphi\left(p\right)} \tag{1}$$

The external boundary has been used as a reference point. The elastic modulus for thickness changes is:

$$\xi = \frac{1}{h}\frac{dh}{dp} \tag{2}$$

Suppose the thickness has a constant elastic modulus. Then:

$$h_n = e^{-\xi(p_e - p)} \tag{3}$$

The normalized thickness is: $h_n = h(p)/h_e$. The pressure, p, in turn depends on the flow rate, the total elasticity of the system and the distance from the well.

Suppose all variables show exponential variation with pressure over a limited pressure range, and then one may define a composite elastic modulus, which is the sum of the individual moduli (Jelmert, 2014). Pressure dependent thickness is the new element in this study.

$$\tau = \gamma + c + \xi - \upsilon = \frac{1}{T_n}\frac{dT_n}{dp} = \frac{1}{T_n}\frac{d\Delta T_n}{d\Delta p} \tag{4}$$

where:

$$T\left(p\right) = \frac{kh}{\mu B} = \frac{k_e e^{\gamma\left(p-p_e\right)} h_e e^{\xi\left(p-p_e\right)}{}_e}{\mu_e e^{\upsilon\left(p-p_e\right)} B_e e^{-c\left(p-p_e\right)}} = \frac{k_e h_e}{\mu_e B_e} e^{\left(\gamma+\xi+c-\upsilon\right)\left(p-p_e\right)} \tag{5}$$

Substitution of eq.(4) into the non-linear flow equation gives a linear diffusivity equation in terms of $T\left(p\right)$. This pressure function, $T\left(p\right)$, may be thought about as transmissibility. All factors are positive. Hence, possible negative values of the pressure-function are unphysical. The linear equation has a well-known solution that can easily be converted back to pressure.

Computation of the well performance index yields:

$$J = \frac{q_{sc}}{\Delta p_{ew}} = \pm \frac{2\pi k_e h}{\mu_e B_e\left(\ln r_{eD} + S_\tau\right)} \cdot \frac{\left(1-e^{-\tau\Delta p_{ew}}\right)}{\tau\Delta p_{ew}} \tag{6}$$

The upper sign is for production, the lower one for injection.

The equivalent equation for homogeneous reservoirs, without stress-sensitivity, is included in the above equation as limiting behaviour. Let $u = \tau\Delta p_{ew}$, then:

$$\lim_{u\to 0}\left\{\frac{\left(1-e^{-u}\right)}{u}\right\} = 1 \tag{7}$$

The above condition may occur for small values of Δp_{ew} and/or τ (i.e. for the $\tau\Delta p_{ew}$-product). Then,

$$J = \frac{\tilde{q}_{sc}}{\Delta p_{ew}} = \pm \frac{2\pi k_e h}{\mu_e B_e\left(\ln r_{eD} + S_\tau\right)} \tag{8}$$

3. Use of the model

To demonstrate the use of the model in an intuitive way, we show the plot from a hypothetical reservoir at the end of the manuscript.

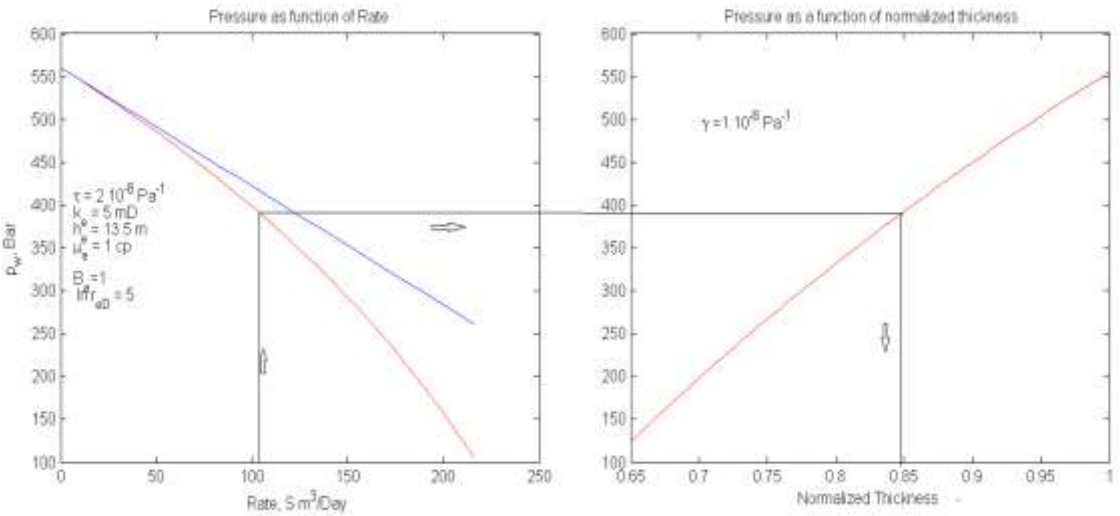

Figure 1: Normalized compaction due to fluid withdrawal.

The left hand plot shows the well performance plot for production, based on eq.(6). The wellbore pressure as a function of normalized thickness, eq.(3), is shown to the right. The red curves are for a drainage area with stress-sensitivity, the blue one without. The use of the plots is illustrated by the arrows. Enter the left hand plot at the rate of interest and follow the arrows to obtain the normalized thickness.

4. Discussion

Steady state flow may not be a realistic assumption. The concept, however, may be useful for engineering calculations. Provided the rate of change in pressure at the external boundary is slow, the flow may be thought about as a series of steady state flow conditions. Then, the reference pressure must be updated whenever a new well performance plot is needed.

From eq.(6) we find that stress-sensitivity is beneficial for injection and detrimental for production. An intuitive explanation is that fracture aperture and thickness tends to decrease with decreasing fluid pressure and increase with increasing pressure. There is a possibility that non-zero elastic moduli may add up to zero eq.(4) and (5). Then, the reservoir is still stress-sensitive. A reservoir is truly without stress-sensitivity when all elastic moduli have zero values.

The different signs for production and injection in eq.(6) leads to different pressure behaviour. This is because the wellbore pressure is located at opposite sides of the reference pressure, p_e.

The pressure function, eq.(5), is not the actual transmissibility, but the best fit to some measured data and as measured with the correlation coefficient. Due to non-unique reservoir responses, many phenomena may give rise to the same response as shown in Fig. 1.

5. Conclusions

A composite elastic modulus, obtained by simple addition, modulus may be useful in well performance predictions. The proposed methodology may be extended to an arbitrary number of quadratic terms. The maximum number is limited by the number of factors in the transport term of the diffusivity equation.

The composite m|odel simplifies to less complex models by assigning zero-value to one or more elastic moduli. In cases characterized by a zero-value or close to zero value composite elastic modulus, it is impossible to distinguish between the well performance of a drainage region with stress-sensitivity from one without.

The proposed model will revert to the conventional model without stress-sensitivity, when all addends in the composite modulus adds up to zero. The reservoir, however, may still be stress-sensitive.

The effect of stress-sensitivity, on rock and fluid properties, should not be overlooked in cases characterized by moduli of high values and/or large pressure changes in the reservoir.

The traditional, homogenous reservoir model without stress-sensitivity is included in the proposed model as limiting behavior. The conventional model may be used with negligible errors for small values of the composite elastic moduli, τ, and/or small pressure changes in the reservoir, Δp_{ew}

For production wells, stress-sensitive permeability has detrimental effect on well performance. For injection wells, it is the other way around.

Nomenclature

B	Formation volume factor
c	Fluid compressibility, Pa^{-1}
T_n	Normalized transmissibility function, given by eq.(5)
ΔT_n	Change in normalized transmissibility from the reference value, $\Delta T_n = 1 - T_n$
h	Thickness, m
J	Productivity/Injectivity index or rate pr. unit pressure change, eq.(6), $Sm^3 s^{-1} Pa^{-1}$
k_e	Permeability at the external boundary, m^2
p	Fluid pressure, Pa
Δp_{ew}	Pressure decrease/increase between external boundary and well,
q_{sc}	Flow rate, Sm^3 / s
$\tilde{q}_{sc}$	Flow rate for a reservoir without stress-sensitivity, eq.(8)
r	Radial distance, m
r_D	Dimensionless distance, $r_D = r / r_w$
r_n	Normalized radial distance, $r_n = r / r_e$

Greek letters

γ	Permeability modulus, Pa^{-1}
τ	Composite modulus, (eq.(4)), Pa^{-1}
υ	Viscosity modulus, Pa^{-1}
ξ	Thickness modulus, Pa^{-1}

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