ANALYTİCAL ANALYSİS OF MEMBRANE BİOREACTOR FİLTRATİON OF HOUSEHOLD WASTEWATER BY EULER, HEUN AND RUNGE-KUTTA METHODS

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ABSTRACT

Environmental scientists have been making use of mathematical models for decades, in order to reduce the time and expense required to interpret and predict the behavior of environmental systems. Mathematical models have particularly being used in order to find out the effects of contributing factors to environmental systems and to be able to produce scenarios by input data manipulation. The aim of this study is to apply three different analytical methods to estimate the COD concentration in the effluent of treated household wastewater by membrane bioreactors, and to see which numerical method provides the best fit to the observed data.

In a previously carried out study (Sarıoğlu, 2011), influent and effluent soluble COD values were determined in greywater, treated by a membrane bioreactor. Table 1 provides the characteristics of data derived from 11 days of experiment.

Table 1. Data characteristics of treated greywater.

Days	x_l Undamaged MLSS mg/L	S_i Soluble COD in influent mg/L	S_e Soluble COD in effluent mg/L
1	350	330.1	66.2
2	500	355.7	68.6
3	450	357	69.3
4	366	246	71.7
5	1010	381.3	33.9
6	1000	149.7	29.1
7	1200	428.9	22.5
8	1152	248.5	21.6
9	1100	207	15.4
10	1128	321.6	14.7
11	1200	311.8	18.3

Following mass balance equation is used to estimate the soluble COD in the effluent.

$$\frac{dS_e}{dt} = \frac{Q}{V}(S_i - S_e) + \frac{q}{V}(S_R - S_e) + \beta k_h x_d - \frac{1}{Y}\frac{\mu_m S_e}{K_s + S_e} x_l$$

where Q is the flowrate (L day^{-1}), V is the aeration tank volume (L), q is the sludge disintegration rate (L day^{-1}), β is the conversion of non-biodegradable particulate to biodegradable substrate mg COD mg $MLSS^{-1}$, μ_m is the maximum specific growth rate (day^{-1}), and K_s is the half-saturation constant mg L^{-1}.

Three different numerical methods, Euler, Heun and Runge-Kutta are applied to estimate the S_e concentrations. In order to achieve this following transformations are carried out for the above equation:

$$\frac{dS_e}{dt} = \frac{Q}{V}(S_i - S_e) + \frac{q}{V}(S_R - S_e) + \beta k_h x_d - \frac{1}{Y}\frac{\mu_m S_e}{K_s + S_e} x_l$$

$$\frac{dS_e}{dt} = \frac{\Delta S_e}{\Delta t} = \frac{S_{e_{i+1}} - S_{e_i}}{t_{i+1} - t_i} = \frac{Q}{V}(S_i - S_e) + \frac{q}{V}(S_R - S_e) + \beta k_h x_d - \frac{1}{Y}\frac{\mu_m S_e}{K_s + S_e} x_l$$

$$S_{e_{i+1}} - S_{e_i} = \left[\frac{Q}{V}(S_i - S_e) + \frac{q}{V}(S_R - S_e) + \beta k_h x_d - \frac{1}{Y}\frac{\mu_m S_e}{K_s + S_e} x_l\right] t_{i+1} - t_i$$

$$S_{e_{i+1}} = S_{e_i} + \left[\frac{Q}{V}(S_i - S_e) + \frac{q}{V}(S_R - S_e) + \beta k_h x_d - \frac{1}{Y}\frac{\mu_m S_e}{K_s + S_e} x_l\right] t_{i+1} - t_i$$

A similar pattern is observed for each method. As can be seen in Fig. 1a, Euler gives a rough approximation to the mass balance equation, while Heun provides a better (Fig. 1b) between t=4 and t=8. Fig. 3b suggests that Runge-Kutta, after t=3, nearly depicts an exact fir to the curve. One reason for this could be the significantly lesser influent COD values observed before day 3. Fig. 1d provides the comparison of three different techniques.

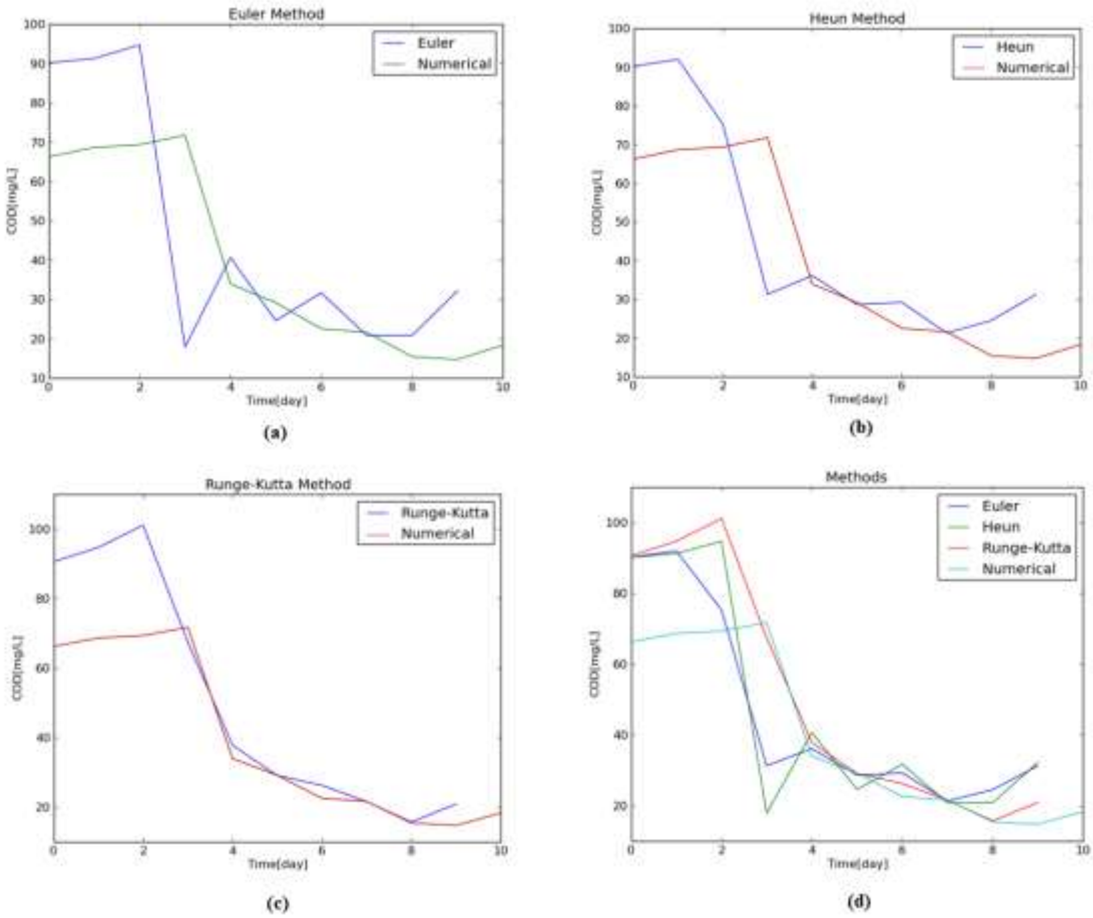

Figure 1. Simulation of soluble COD in effluent (a) Euler method (b) Heun method (c) Runge-Kutta method (d) Comparison of methods.

Mathematical techniques such as, Euler, Heun and Runge-Kutta methods are commonly applied as numerical solutions to various environmental problems. Analysis results suggest that

Runge-Kutta technique provided the best fit amongst the other techniques such as Euler and Heun.

Keywords: Numerical Methods, Euler, Heun, Runge-Kutta, Greywater, COD.

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