AQUIFER DRAWDOWN AND VARIABLE-STAGE STREAM DEPLETION INDUCED BY A NEARBY PUMPING WELL

BAALOUSHA H.M.

Qatar Environment & Energy Research Institute (QEERI), PO Box: 5825,
Doha Qatar
E-mail: Baalousha@web.de

ABSTRACT

A stream depletion phenomenon has been studied for many decades, and different analytical models were developed to find the effect of a pumping well on a nearby stream. Most developed models consider a constant stream stage or neglect the variation in stream stage. This is not the case in reality where the streams flow and level continuously vary over time.

In this paper a new analytical model was developed considering variation in stream flow (i.e. stream stage). The developed model considers the recession of stream flow and its impact on stream depletion and drawdown. Comparison between the developed solution and the existing ones shows a significant discrepancy when the stream flow varies.

Keywords: Stream depletion, stream recession, analytical solution, drawdown.

1. Introduction

Numerous studies were conducted to understand the phenomenon of the stream depletion resulting from a pumping well. Theis (1941) was the first to study the effect of a pumping well on a flowing stream producing a simple solution, which was later developed by Glover and Balmer (1954). The latter model was further developed by Hantush (1968) to account for streambed conductance. Hunt (1999) modified the Hantush solution to better represent the effect of partially clogged streambed on stream depletion and drawdown. Baalousha (2012) derived the same solution of Hunt (1999), based on a superposition approach and considering a stream of a finite width.

In all solutions developed so far it was assumed the stream level (i.e. stage) is constant all the time. However, this contradicts the reality that stream flow and stage vary continuously. This paper develops a solution that accounts for stream level variation over time and incorporates this variation in analysis of stream depletion and drawdown.

2. Mathematical Development

2.1. Drawdown

Baalousha 2012 derived the drawdown in aquifer resulting from a pumping well near a stream based on superposition of a pumping well (s_w) and a losing stream (s_r). Considering a coordinate system at the centre of a stream, the well drawdown (s_w) at any point (x,y) and at any time t is given by Theis Equation:

$$s_w(x,y,t) = \frac{Q}{4\pi T} Ei\left(-\frac{S((L-|x|)^2 + y^2}{4Tt}\right) \tag{1}$$

where Q is the pumping rate, S is the aquifer storativity, L is the distance between a pumping well and the stream, t is time since pumping starts and T is the aquifer transmissivity. Ei is the exponential integral (also known as well function). The drawdown resulting from the stream alone s_r is given by (Hunt 1999, Baalousha, 2012):

$$s_r(x,y,t) = \frac{Q\lambda}{8\pi T^2}\int_0^{\infty} Ei\left(-\frac{S((L+|x|+\eta)^2+y^2)}{4Tt}\right)e^{-\lambda\eta/2T}d\eta \tag{2}$$

Where λ is the streambed conductance [m^2/T]. Equation (2) is based on the assumption of a constant stream level, as is the case in all developed analytical solutions. Assuming *$\Phi(t)$* is the function of stream level variation over time then s_v is given using the convolution integral (Carslaw and Jaeger, 1959):

$$s_v(x,y,t) = \int_0^t \Phi(\tau)\frac{\partial}{\partial t}s_r(x,y,t-\tau)d\tau \tag{3}$$

Where s_v is the aquifer drawdown resulting from a variable stage stream leakage. From (2) and (3):

$$s_v(x,y,t) = \frac{Q\lambda}{8\pi T^2}\int_0^t \frac{\Phi(\tau)}{t-\tau}\int_0^{\infty} e^{-\lambda\eta/2T}\cdot e^{\left(-S\frac{((\eta+L+|x|)^2+y^2)}{4T(t-\tau)}\right)}d\eta d\tau \tag{4}$$

Integrating Equation (4) with respect to η yields:

$$s_v(x,y,t) = \frac{Q\lambda}{8\pi^{1/2}T^{3/2}S^{1/2}}\int_0^t \frac{\Phi(\tau)}{(t-\tau)^{1/2}}e^{\left(\frac{\lambda^2(t-\tau)+2\lambda S(L+|x|)-S^2y^2/(t-\tau)}{4ST}\right)}erfc\left(\frac{S(L+|x|)+\lambda(t-\tau)}{\sqrt{4ST(t-\tau)}}\right)d\tau \tag{5}$$

If the stream stage is constant, then according to assumptions of previous solutions given in Equation (2) Φ=1. This is because it was assumed in Hunt's (1999) solution that the pre-pumping groundwater level and the river are at the same level. In this case, Equations (4) and (2) become identical.

2.2. Stream Depletion

Based on Darcy's Law, and following the same approach of Baalousha (2012), total stream depletion (q_r) is given by:

$$q_r = -2T\int_{-\infty}^{\infty}\left(\frac{\partial s_v}{\partial x}\right)_{x=0}dy \tag{6}$$

Equation (6) is multiplied by 2 because stream depletion occurs at both sides of the stream. Equation (6) can be further simplified as:

$$q_r = -4T\int_0^{\infty}\left(\frac{\partial s_v}{\partial x}\right)_{x=0}dy \tag{7}$$

Because the s_v function is symmetric at both sides of x-axis (i.e. s_y does not change when *y* sign changes, as shown in Equation (5)), and *$\Phi(t)$* is a function of *t* only. Based on Equation (5), the flow gradient at the edge of the stream can be written as:

$$\begin{aligned}\left(\frac{\partial s_v}{\partial x}\right)_{x=0} &= \frac{Q\lambda^2}{16\pi^{1/2}T^{5/2}S^{1/2}}\int_0^t \frac{\Phi(\tau)}{(t-\tau)^{1/2}}erfc\left(\frac{SL+\lambda(t-\tau)}{\sqrt{4ST(t-\tau)}}\right)e^{\left(\frac{\lambda^2(t-\tau)+2\lambda LS-S^2y^2/(t-\tau)}{4ST}\right)}d\tau - \\ &\frac{Q\lambda}{8\pi T^2}\int_0^t \frac{\Phi(\tau)}{(t-\tau)}e^{\left(-\frac{(S^2L^2+S^2y^2)}{4ST(t-\tau)}\right)}d\tau\end{aligned} \tag{8}$$

From (7) and (8) the stream depletion is:

$$q_r = \frac{Q\lambda}{2\pi^{1/2}T^{1/2}S^{1/2}} \int_0^t \frac{\Phi(\tau)}{(t-\tau)^{1/2}} e^{\left(-\frac{S^2L^2}{4ST(t-\tau)}\right)} d\tau - \frac{Q\lambda^2}{4TS} \int_0^t \Phi(\tau) erfc\left(\frac{SL+\lambda(t-\tau)}{\sqrt{4ST(t-\tau)}}\right) e^{\left(\frac{\lambda^2(t-\tau)+2\lambda LS}{4ST}\right)} d\tau \quad (9)$$

2.3. Example

An example is used to demonstrate the effect of pumping on drawdown and stream depletion when the stage varies over time. This example assumes an exponential change of stream level, which is consistent with a typical recession curve (Tallaksen, 1995, Toebes 1964). In this case, the stream stage function (s_t) is:

$$s_t = s_0 e^{-at} \quad (10)$$

Where s_0 is the initial stream level, *a* is a constant and *t* is time. Using Equation (5) and (10), the drawdown is:

$$s_v(x,y,t) = \frac{Q\lambda}{8\pi^{1/2}T^{3/2}S^{1/2}} \int_0^t \frac{s_0 e^{-at}}{(t-\tau)^{1/2}} e^{\left(\frac{\lambda^2(t-\tau)+2\lambda S(L+|x|)-S^2y^2/(t-\tau)}{4ST}\right)} erfc\left(\frac{S(L+|x|)+\lambda(t-\tau)}{\sqrt{4ST(t-\tau)}}\right) d\tau \quad (11)$$

Based on Equations (9) and (10), the stream depletion is:

$$q_r = -\frac{Q\lambda^2}{4TS} \int_0^t s_0 e^{-at} erfc\left(\frac{SL+\lambda(t-\tau)}{\sqrt{4ST(t-\tau)}}\right) e^{\left(\frac{\lambda^2(t-\tau)+2\lambda LS}{4ST}\right)} d\tau + \frac{Q\lambda}{2\pi^{1/2}T^{1/2}S^{1/2}} \int_0^t \frac{s_0 e^{-at}}{(t-\tau)^{1/2}} e^{\left(-\frac{S^2L^2}{4ST(t-\tau)}\right)} d\tau \quad (12)$$

It is assumed that a pumping well is located at a distance L=300 meters from the stream centreline. The stream width is 10 meters and the pumping rate is 1000 m3/day. Storativity (S) and transmissivity (T) were assumed to be 0.04 and 400 m^2/day, respectively and the vertical hydraulic conductivity of the river bed is 0.1. Figure 1 shows the total drawdown over time resulting from stream leakage for both cases of variable-stage stream and constant-stage stream. The total drawdown is the sum of ground water level rise due to stream leakage and drop due to pumping. It is given by:

$$s_v(x,y,t) = \frac{Q\lambda}{8\pi^{1/2}T^{3/2}S^{1/2}} \int_0^t \frac{s_0 e^{-at}}{(t-\tau)^{1/2}} e^{\left(\frac{\lambda^2(t-\tau)+2\lambda S(L+|x|)-S^2y^2/(t-\tau)}{4ST}\right)} erfc\left(\frac{S(L+|x|)+\lambda(t-\tau)}{\sqrt{4ST(t-\tau)}}\right) d\tau - \frac{Q}{4\pi T} Ei\left(-\frac{S(x^2+y^2)}{4Tt}\right) \quad (13)$$

Both constant and variable stage have similar drawdown in the very early time but then the variable stage shows higher drawdown (i.e. less leakage) due to declining stage over time, as given by Equation (10).

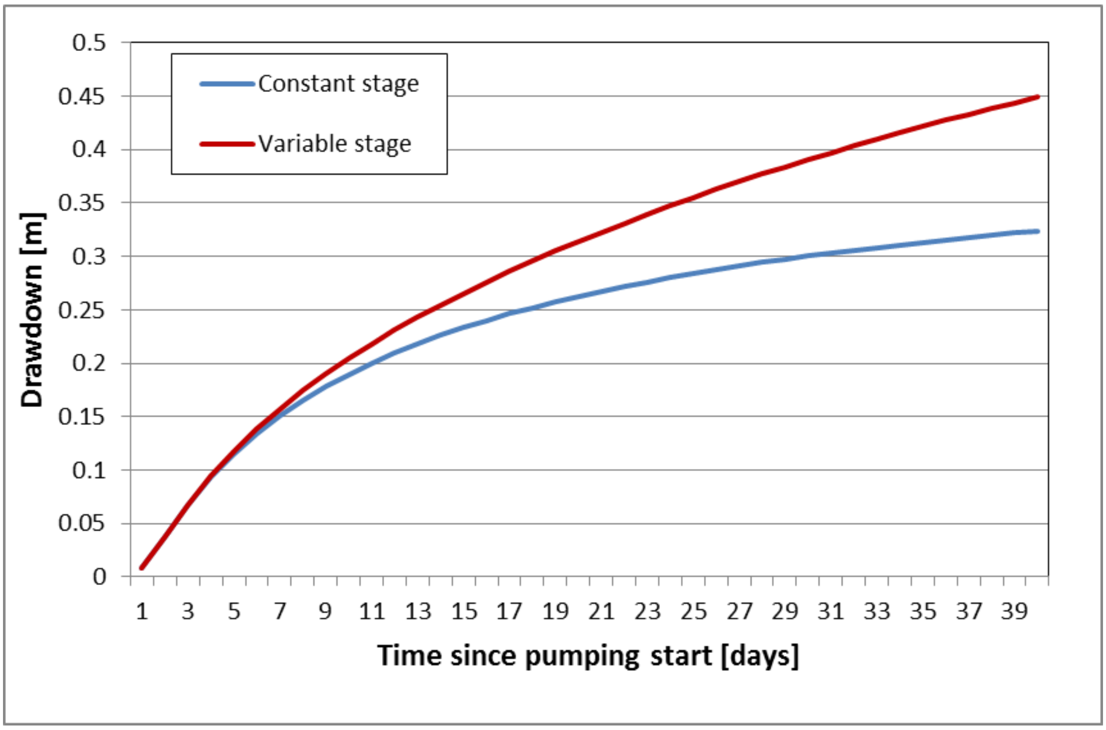

Figure 1: Drawdown at the edge of the stream for both constant and variable stage**.**

The dimensionless stream depletion (qr/Q) resulting from the pumping well is shown in Figure 2. The variable-stage stream shows a considerably less depletion (i.e. flow to aquifer), compared to the constant-stage stream. This is because the stage of the stream decreases over time at faster rate of declining water table by pumping. As a consequence, the head difference between the stream and the aquifer reduces, which reduces stream depletion.

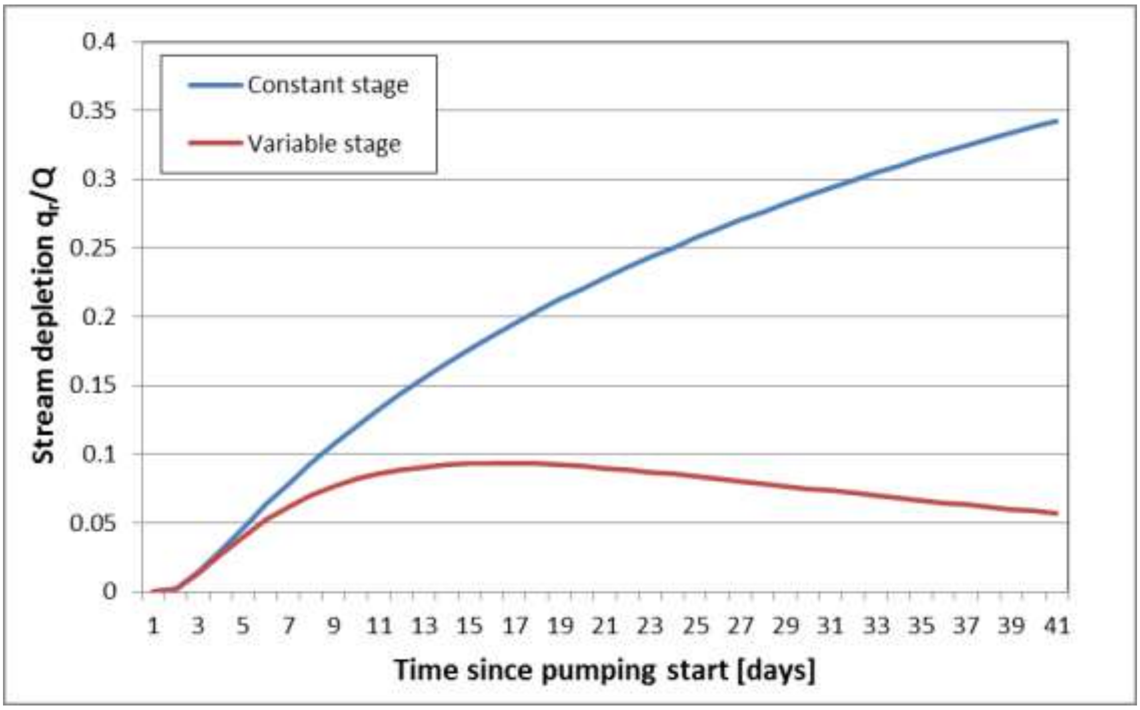

Figure 2: Stream depletion for both constant and variable stage.

3. Conclusion

A new solution has been derived in this study to calculate the drawdown and stream depletion resulting from a nearby pumping well. This solution is more generalised than existing solutions in the literature, as it considers variation in stream stage. As shown in this study, changes in stream stage significantly affect both drawdown and stream depletion. It is found that both drawdown and stream depletion decrease over time as a result of stage recession.

The solution presented can be used with any function (or constant) representing the changes in stream stage over time. It also can be used with supersposition in case the stage function changes over time.

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