THE EFFECT OF SHAPE FACTOR ON THE AVERAGE BED SHEAR STRESS IN OPEN CHANNEL FLOW

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ABSTRACT

This paper presents an analytical method to determine the effect of side wall correction factor on the sediment transport. The analysis is based on the computation of boundary shear stress distributions acting on the flow perimeter of prismatic open channels. The application depends on the principle of steady and uniform flow in smooth trapezoidal open channels with different side slopes. The average bed shear stress is calculating by utilizing Darcy-Weisbach, Manning and Chézy roughness coefficients. Based on this concept, a novel method is developed for the partitioning of the flow cross-sectional area into various parts corresponding to the channel shape and roughness composition on the wetted perimeter. The technique permits the distribution of the boundary shear over the wetted perimeter to be evaluated within each sub-flow area. Analytical equations governing the boundary shear stress distributions have been derived and are valid for all channel aspect ratios. The final equation was applied to calculate boundary shear distribution in trapezoidal open channels. The results show that as the channel side slope θ increases, the friction factor of the bed will increase. Consequently, both the bed shear stress τ_b and sediment transport will also increase.

Keywords: bed shear stress, sediment transport, side wall correction factor, open channel

1. Introduction

The shape of the channel cross section is one of the factors influencing the bed load function. If this section is not influenced either structurally or by vegetation it is only a function of the sediment and of the flow, Einstein (1950). The shape factor for the reach must enter into the analysis of natural river because the shape factor affects the energy losses. This is due to bends and banks and the effect of the shape factor on the velocity distribution, causing variation in velocity, width, depth, boundary shear and secondary circulation, Simons and Sentürk, 1992.

The cross section will be more trapezoidal or rectangular in a crossing. Cross-section shape can be described by a number of variables, such as the area, width, and maximum depth. In wide channels with W/d greater than approximately 20, the hydraulic radius and the mean depth are approximately equal. The conveyance or capacity of a channel is related to the area and hydraulic radius and is defined as $AR^{2/3}$ (Watson *et al*., 1999).

Bagnold's (1980) research implies that for a given discharge and slope, larger sediment load requires a wider channel. Whereas Henderson's (1966) threshold channel equations imply the contrary, sediment transport increases as depth increases; therefore, the width/depth ratio decreases. A resolution of these arguments is achieved by the extremal hypothesis and comparisons made by White *et al*. (1982) show that the maximization of sediment discharge and minimization of stream power, or slope for given water discharge, will give the same result if sediment discharge is a variable that can be maximized. proposed by White *et al*. (1982), in which maximum transporting capacity is attained at some intermediate width. This hypothesis was applied to the prediction of hydraulic and geometric characteristics of alluvial channels with sand and gravel beds. The analyses

The wall friction factor f_w for turbulent flow over smooth boundary $10^5 < \mathrm{Re}/f < 10^8$ can be calculated from (Julien, 1998):

$$f_w = 0.0026\left(\log\left(\frac{\mathrm{Re}}{f}\right)\right)^2 - 0.0428\log\left(\frac{\mathrm{Re}}{f}\right) + 0.1884$$

The Reynolds number is calculated from the average velocity $V_{avg} = \dfrac{Q}{wd}$

$$\mathrm{Re} = \frac{4R_h V_{avg}}{v}$$

where v is the kinetic viscosity of the fluid.

The Chézy formula describes the mean flow velocity of steady, turbulent open channel flow. The Chézy equation is derived from hydrodynamics theory. Unlike The Manning formula is an empirical formula estimating the average velocity of a liquid flowing in open channels driven by gravity. Darcy equation is derived based on the equilibrium of forces in the direction of flow for uniform and steady state flow. These equations are well-known equations that utilized to compute the flow discharge and flow velocity in open channel. These equations dependent on many factors, including surface roughness, longitudinal slope and channel cross sectional geometry. These equations are used in this research to analyze the effect of shape factor on the average bed shear stress in open channel flow.

2. Present Approach

To calculate the average bed shear stress Einstein assumed that the cross section can be divided into separate subsections with each subsection having the same energy line slope, and the same average flow velocity and the flow formula can be applied to each subsection individually.

The total resistance to flow can be described in term of the Chézy coefficient C, the Darcy-Weisbach friction factor f, or the Manning coefficient n. The following identity between three factors has been established Julien (1998):

$$C = \sqrt{\frac{8g}{f}} = \frac{R_h^{1/6}}{n} \text{ (in SI units)}$$

$$= \frac{1.49 R_h^{1/6}}{n} \text{ (in English units)}$$

where,

R_h is the hydraulic radius

g is the gravitational acceleration

The fundamental dimensions are as follows:

C is in $L^{1/2}/T$, f is dimensionless , and n is in $T/L^{1/3}$.

To calculate the average bed shear stress, τ_b, it is necessary to calculate the hydraulic radius, R_b.

Where, $\tau_b = \rho \times g \times R_b \times S$

ρ is the flow density, S is the channel longitudinal slope , and R_b is the hydraulic radius related to the channel bed

By using the Darcy-Weisbach, the Manning or the Chézy equation, as the flow formula, the average friction factor of the total cross section can be calculated. Analysis the effect of changing cross sectional geometry such as width depth ratio and the inclination of side slope on the sediment transport, using Darcy-Weisbach, Manning and Chézy equations are presented in this research.

3. Analysis and Results

3.1. The Darcy-Weisbach equation

$$f_{avg} = \frac{8gR_{h\,avg}S_f}{V^2_{avg}} \tag{1}$$

where $R_{h\,avg}$ is the hydraulic radius of the total cross section and is equal to $\frac{A}{P}$, A is the total area of the cross section, and P is the wetted perimeter of the total cross section.

where,

$$P = 2d + w$$

d is the flow depth, and w is the channel width

Applying the flow formula to each subsection yields:

$$\frac{V^2_{avg}}{8gS_f} = \frac{R_{h\,i}}{f_i} = \frac{R_{h\,avg}}{f_{avg}} \tag{2}$$

where R_i is the hydraulic radius of subsection i, and f_i is the friction factor of subsection i.

The total area of the cross section, A, is the sum of the area of n subsection, A_i, hence

$$A = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} R_{h\,i}P_i = \frac{V^2_{avg}}{8gS_f}\sum_{i=1}^{n} f_i P_i \tag{3}$$

The average friction factor, f_{avg}, can be written in the form:

$$f_{avg} = \frac{\sum_{i=1}^{n} f_i P_i}{\sum_{i=1}^{n} P_i} \tag{4}$$

For rectangular cross section

$$f_{avg} = \frac{f_b \times w + f_w \times 2d}{w + 2d} \tag{5}$$

where f_{avg} is the average friction factor for the total cross section, f_b is the friction factor of the bed , f_w is the friction factor of the wall, w is the channel width , and d is the channel depth.
Arranging the above equation yields,

$$f_b = f_{avg} + \frac{2d}{w}\left(f_{avg} - f_w\right) \tag{6}$$

Introducing a non-dimensional channel shape factor width-depth ratio $\psi = \frac{w}{d}$

$$f_b = f_{avg} + \frac{2}{\psi}\left(f_{avg} - f_w\right) \tag{7}$$

As the shape factor ψ increases the friction factor of the bed decreases, because $f_b = \frac{\tau_b}{\frac{1}{8}\rho V^2{}_{avg}}$, then both the bed shear stress τ_b and sediment transport will decrease.

For trapezoidal cross section with side slope θ measured from the vertical, the friction factor of the bed, f_b is:

$$f_b = f_{avg} + \frac{2}{\psi\cos\theta}\left(f_{avg} - f_w\right) \tag{8}$$

As the channel side slope θ increases the friction factor of the bed increases. Therefore, both the bed shear stress τ_b and sediment transport will increase.

3.2. The Manning Equation

$$V_i = \frac{1}{n_i} R_{h\,i}{}^{2/3} S_f{}^{1/2} \text{(in SI units)} \tag{9}$$

$$\frac{V_{avg}}{S_f{}^{1/2}} = \frac{R_{hi}{}^{2/3}}{n_i} = \frac{R_{avg}{}^{2/3}}{n_{avg}} \tag{10}$$

$$R_{h\,i} = \frac{V_i^{3/2} n_i^{3/2}}{S_f{}^{3/4}} \tag{11}$$

$$A = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} R_{h\,i} P_i = \frac{V^{3/2}{}_{avg}}{S_f{}^{3/4}} \sum_{i=1}^{n} n_i^{3/2} P_i \tag{12}$$

$$n_{avg} = \frac{R_{h\,avg}{}^{2/3} Sf^{1/2}}{V_{avg}} \tag{13}$$

The average Manning coefficient, n_{avg} , can be written in the form:

$$n_{avg} = \left[\frac{\sum_{i=1}^{n} n_i^{3/2} P_i}{\sum_{i=1}^{n} P_i} \right]^{2/3} \quad (14)$$

For rectangular cross section

$$n_{avg} = \left[\frac{n_b^{3/2} \times w + n_w^{3/2} \times 2d}{w + 2d} \right]^{2/3} \quad (15)$$

where n_{avg} is the Manning coefficient for the total cross section, n_b is the Manning coefficient of the bed , n_w is the Manning coefficient of the wall, w is the channel width and d is the channel depth. Arranging the above equation yields

$$n_b^{3/2} = n_{avg}^{3/2} + \frac{2}{\psi} \left(n_{avg}^{3/2} - n_w^{3/2} \right) \quad (16)$$

As the shape factor ψ increases the Manning coefficient of the bed n_b decreases, because $\tau_b = \rho g R_b S_f$, $R_{hb}^{2/3} S_f^{1/2} = V \times n_b$, if n_b decreases then the value of $R^{2/3} S_f^{1/2}$ will decrease, consequently, τ_b will decrease, and sediment transport will decrease.
For trapezoidal cross section with side slope θ measured from the vertical, the Manning coefficient of the bed n_b is

$$n_b^{3/2} = n_{avg}^{3/2} + \frac{2}{\psi \cos\theta} \left(n_{avg}^{3/2} - n_w^{3/2} \right) \quad (17)$$

As the channel side slope θ increases the Manning coefficient of the bed increases, both τ_b and sediment transport will increase.

3.3. The Chézy Equation

$$V_i = C_i R_{hi}^{1/2} S_f^{1/2} \quad (18)$$

$$\frac{V_{avg}}{S_f^{1/2}} = C_i R_{hi}^{1/2} = C_{avg} R_{havg}^{1/2} \quad (19)$$

$$R_{hi} = \frac{V_i^2}{C_i^2 S_f} \quad (20)$$

$$A = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} R_{hi} P_i = \frac{V^2{}_{avg}}{S_f} \sum_{i=1}^{n} \frac{P_i}{C_i^2} = C_{ave}^2 R_{havg} \sum_{i=1}^{n} \frac{P_i}{C_i^2} \quad (21)$$

The average Chézy coefficient, C_{avg} , can be written in the form:

$$C_{avg} = \left[\frac{\sum_{i=1}^{n} P_i}{\sum_{i=1}^{n} \frac{P_i}{C_i^{\,2}}} \right]^{1/2} \tag{22}$$

For rectangular cross section

$$C_{avg} = \left[\frac{w + 2d}{\left(\frac{w}{C_b^{\,2}} + \frac{2d}{C_w^{\,2}} \right)} \right]^{1/2} \tag{23}$$

where C_{avg} is the Chézy coefficient for the total cross section, C_b is the Chézy coefficient of the bed , C_w is the Chézy coefficient of the wall, w is the channel width ,and d is the channel depth. Arranging the above equation yields

$$C_{avg}^{\,2} \left(\frac{w}{C_b^{\,2}} + \frac{2d}{C_w^{\,2}} \right) = w + 2d \tag{24}$$

$$\left(\frac{C_{avg}^{\,2} \times w}{C_b^{\,2}} \right) = w + 2d - \frac{C_{avg}^{\,2} \times 2d}{C_w^{\,2}} = \frac{C_w^{\,2}(w + 2d) - C_{avg}^{\,2} \times 2d}{C_w^{\,2}} \tag{25}$$

$$\frac{1}{C_b^{\,2}} = \frac{C_w^{\,2}(w + 2d) - C_{avg}^{\,2} \times 2d}{C_w^{\,2} \times C_{avg}^{\,2} \times w} = \frac{1}{C_{avg}^{\,2}} + \frac{2}{\psi C_{avg}^{\,2}} - \frac{2}{\psi C_w^{\,2}} \tag{26}$$

$$C_b = \left[\frac{1}{\frac{1}{C_{avg}^{\,2}} + \frac{2}{\psi} \left(\frac{1}{C_{avg}^{\,2}} - \frac{1}{C_w^{\,2}} \right)} \right]^{1/2} \tag{27}$$

As the shape factor ψ increases the Chézy coefficient of the bed C_b increases. Because $\tau_b = \rho g R_b S_f$, $R_{hb}^{\,1/2} S_f^{\,1/2} = \frac{V}{C_b}$, if C_b increases, the value of $R^{1/2} S_f^{\,1/2}$ will decreaseas a result, both τ_b and sediment transport will decrease.

For trapezoidal cross section with side slope θ measured from the vertical, the Chézy coefficient of the bed C_b is

$$C_b = \left[\frac{1}{\frac{1}{C_{avg}{}^2} + \frac{2}{\psi \cos\theta}\left(\frac{1}{C_{avg}{}^2} - \frac{1}{C_w{}^2} \right)} \right]^{1/2} \quad (28)$$

As the channel side slope θ increases the Chézy coefficient of the bed C_b decreases, subsequently, τ_b and sediment transport will increase.

4. Conclusion

For rectangular cross section in open channel flow it is found that as the shape factor ψ increases both the bed shear stress τ_b and sediment transport will decrease. For trapezoidal cross section as the channel side bank slope θ increases, the friction factor of the bed increases then both the bed shear stress τ_b and sediment transport will increase.

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